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## Recent Results from SLR Experiments in Fundamental Physics: Frame Dragging observed with Satellite Laser Ranging.

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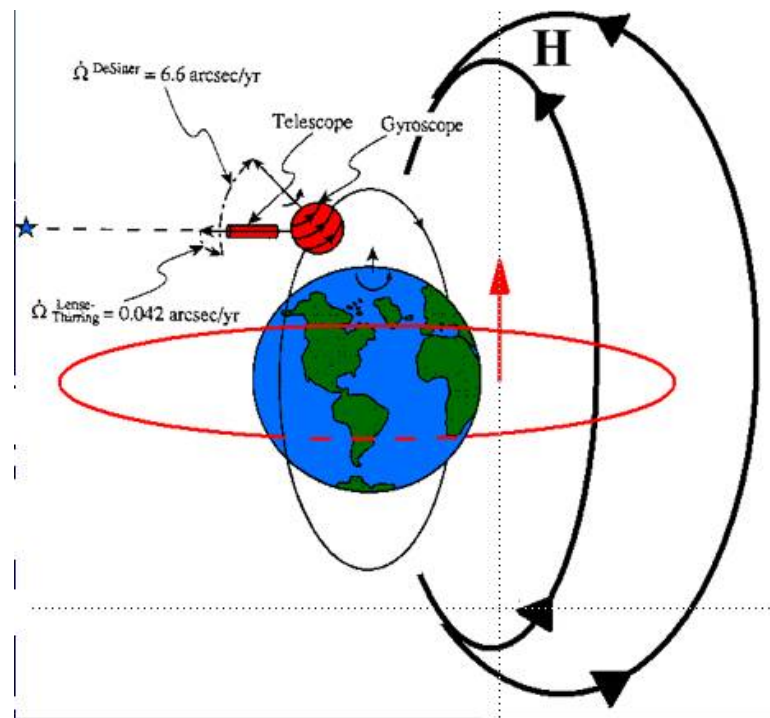
### Abstract

*Satellite laser ranging provided for decades the most precise measurement of positions and velocities of earthbound tracking stations, as well as the most precise orbits of earth-orbiting artificial satellites. While the latter applies to any satellite carrying the appropriate reflectors, the use of these orbits for precise geodetic products requires the use of specially designed target satellites in high altitude orbits, such as the two LAGEOS satellites. To achieve such high quality, the motion of these satellites must be described with equally accurate models, such as those made available recently, thanks to missions like CHAMP and GRACE. This led to the synergistic application of such precise products to devise tests of fundamental physics theories. Nearly twenty years after conceiving and proposing an initial concept for a General Relativity (GR) prediction test, our recent experiment resulted in a positive and convincing measurement of the Lense-Thirring effect, also known as the gravitomagnetic effect of the rotating Earth. Using state-of-the-art Earth gravitational field models based on data from the CHAMP and GRACE missions, we obtained an accurate measurement of the Lense-Thirring effect predicted by GR, analyzing eleven years of LAGEOS and LAGEOS 2 Satellite Laser Ranging (SLR) data. The new result, in agreement with the earlier one based on Earth models JGM-3 and EGM96, is far more accurate and more robust. The present analysis uses only the nodal rates of the two satellites, making NO use of the perigee rate, thus eliminating the dependence on this unreliable element. Using the EIGEN-GRACE02S model, we obtained our optimal result:  $\mu = 0.99$  (vs. 1.0 in GR), with a total error between  $\pm 0.05$  and  $\pm 0.1$ , i.e., between 5% and 10 % of the GR prediction. Results based on processing with NASA and GFZ s/w will be presented, along with preliminary tests with very recent improved GRACE models. Further improvement of the gravitational models in the near future will lead to even more accurate tests. We discuss the LAGEOS results and some of the crucial areas to be considered in designing the future LARES mission dedicated to this test.*

### Introduction

One of the most fascinating theoretical predictions of general relativity is “frame-dragging” (Misner et al. 1973, Ciufolini and Wheeler 1995), also known as the Lense-Thirring effect, after the two Austrian physicists who predicted the effect based on Einstein’s General Relativity (GR) theory (Lense and Thirring, 1918). The equivalence principle, at the basis of Einstein’s gravitational theory, states that “locally”, in a sufficiently small spacetime neighbourhood, in a freely falling frame, the observed laws of physics are the laws of special relativity. However, the axes of these inertial frames where “locally” the gravitational field is “unobservable”, rotate with respect to “distant stars” due to the rotation of a mass or in general due to a

current of mass–energy. In general relativity the axes of a local inertial frame can be realized by small gyroscopes, as shown in Figure 1.



**Figure 1.** The gravitomagnetic field and the mass-energy currents that produce the frame-dragging effect on the node of the orbiting gyroscope.

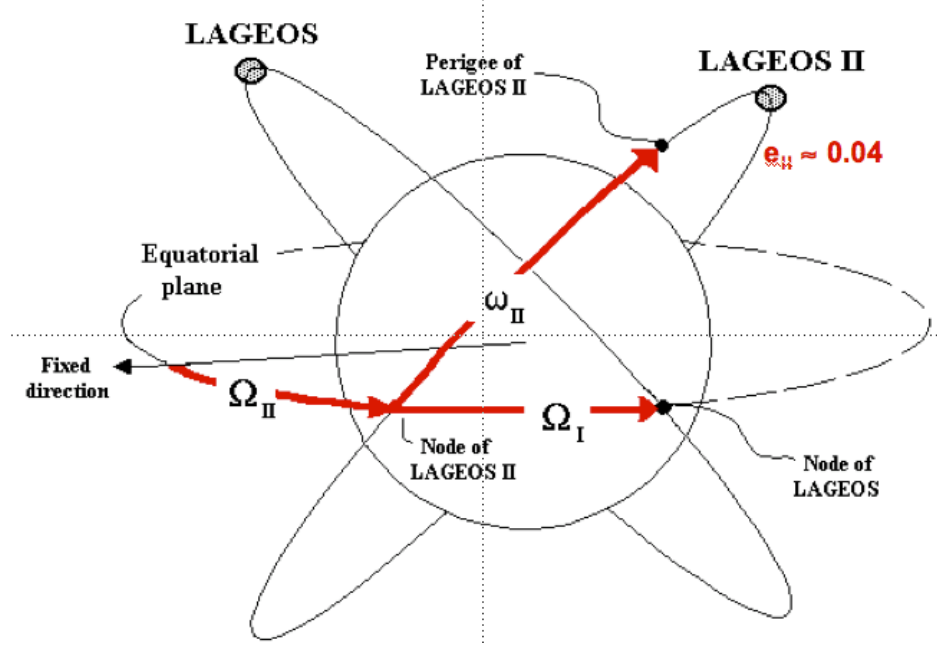
## Methodology

The gravitomagnetic force is by far smaller than the gravitational monopole, so we can use the tools of celestial mechanics and consider this force as a perturbation on an orbiting satellite. From the integrated (to first order) perturbation equations we obtain the most significant effects on the orbital elements, the secular rates of the node and perigee:

$$\left\{ \begin{array}{l} \dot{\Omega}^{L-T} = \frac{2GJ}{c^2 a^3 (1-e^2)^{3/2}} \\ \dot{\omega}^{L-T} = \frac{-6GJ}{c^2 a^3 (1-e^2)^{3/2}} \cos I \end{array} \right.$$

In the past we used both quantities in our methodology (Ciufolini *et al.*, 1998) due to the lack of accurate enough gravitational models. Since the release of improved models from the CHAMP and GRACE missions though, we only use the node rate in our experiments. Our methodology uses as “source” of the field Earth with its angular momentum, as a test particle the geodetic satellites LAGEOS and LAGEOS 2 at present (and in the future LARES, see more on this later), and our basic observations are the two-way precise ranging with laser pulses from the ground network of the International Laser Ranging Service (ILRS), (Pearlman *et al.*, 2002).

Perturbations due to  $J_2$  are much larger than the Lense-Thirring (LT) effect, so we need to be able to eliminate such uncertainties in order to extract the sought-for LT signal from our data. Thanks to Ciufolini's 1986 idea however, (using a "butterfly" configuration of counter-orbiting satellites in supplementary inclination orbits, Figure 2), the effect of  $J_2$  uncertainties is cancelled.



**Figure 2.** The nearly-“butterfly” configuration of the retrograde LAGEOS ( $i = 109.8^\circ$ ) and the prograde LAGEOS 2 ( $i = 52.6^\circ$ ) orbits.

When the two orbits are supplementary, one-half the sum of their nodal rate variations would provide a direct observation of the LT effect. However, Ciufolini (1989) generalized his original idea of the butterfly configuration to configurations of  $N$  nodes of various orbits, to cancel out the effects of the first  $N-1$  even zonals on the nodal rates of these orbits. Using this modified constraint for the case of two orbits in near- (but not exact) butterfly configuration, such as the LAGEOS and LAGEOS 2 orbits, we obtain:

$$\delta\dot{\Omega}_I + k\delta\dot{\Omega}_{II} = 48.2\mu + \text{other errors} \quad [\text{mas/y}]$$

where  $k$  ( $\approx 1/2$ ) is a function of the elements of the two orbits, and  $\mu$  is our LT parameter to be determined. If  $\mu = 1$ , GR is correct, if  $\mu = 0$  the Newtonian physics are correct. Under “other errors” we lump a number of higher order errors and the uncertainty in the background models mapped on the estimated quantity  $\mu$ . Extensive error analysis of the experiment provides bounds on these errors and allows for a realistic error budget for the result (Ciufolini, Pavlis and Peron, 2006). We separate the error sources in two groups, the gravitational and the non-gravitational. A summary of the results published in detail in (*ibid.*) are given in Figures 3 and 4.

This study supports the errors quoted for our most recent published results for  $\mu$ , (Ciufolini and Pavlis, 2004), between 5 and 10% of the expected value of 1 for GR. This improved (in accuracy) result compared to our 1998 result, is a direct consequence of the highly improved gravitational model accuracy, thanks to the use

of gravity mapping data from the CHAMP and GRACE missions (Reigber *et al.*, 2002, 2003, 2005 and Tapley *et al.*, 2002 and 2003). These products are the enabling factors for the success of these experiments. Pavlis (2002) and Ries *et al.* (2003) had already forewarned of this leap in accuracy for these models and proposed the continuation of the LAGEOS experiments in anticipation of their release.

### Gravitational perturbations:

- Even zonal harmonic coefficients  $J_{2n}$  of the geopotential (static part)
- Odd zonal harmonic coefficients  $J_{2n+1}$  (static part)
- Non zonal harmonic coefficients (Tesseral and Sectorial)
- Solid and ocean Earth tides and other temporal variations of Earth gravity field
- Solar, lunar and planetary perturbations
- de Sitter precession
- Other general relativistic effects
- Deviations from geodesic motion

$$\delta\mu^{\text{even zonals}} \leq 3\text{-}4\% \mu^{\text{GR}}$$

$$\delta\mu^{\text{odd zonals}} \leq 10^{-3} \mu^{\text{GR}}$$

$$\delta\mu^{\text{tides}} \leq 1\% \mu^{\text{GR}}$$

$$\delta\mu^{\text{other ...}} \leq 10^{-3} \mu^{\text{GR}}$$

*Figure 3. The calibrated errors on  $\mu$ , due to realistic uncertainties of the gravitational parameters.*

### Non-gravitational perturbations:

- Solar radiation pressure
- Earth albedo
- Anisotropic emission of thermal radiation due to Sun visible radiation (Yarkovsky-Schach effect)
- Anisotropic emission of thermal radiation due to Earth infrared radiation (Yarkovsky-Rubincam effect)
- Neutral and charged particle drag
- Earth magnetic field

$$\delta\mu^{\text{solar rad.}} \leq 10^{-3} \mu^{\text{GR}}$$

$$\delta\mu^{\text{albedo}} \leq 1\% \mu^{\text{GR}}$$

$$\delta\mu^{\text{Y-S}} \leq 1\% \mu^{\text{GR}}$$

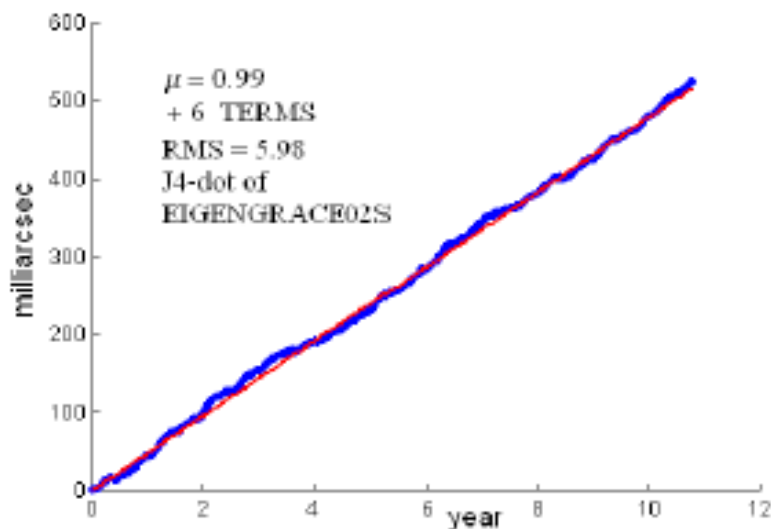
$$\delta\mu^{\text{Y-R}} \leq 1\% \mu^{\text{GR}}$$

$$\delta\mu^{\text{Drag-like}} \leq 10^{-3} \mu^{\text{GR}}$$

*Figure 4. The calibrated errors on  $\mu$ , due to realistic uncertainties of the non-gravitational parameters.*

## The 2004 experiment results

The most accurate results on the measurement of the LT effect were published in (Ciufolini and Pavlis, 2004). The methodology and error analysis were subsequently detailed in (Ciufolini, Pavlis and Peron, 2006). These two references describe in detail the technique and the data that were used for the 2004 experiment. The basic points to be noted here are that the analysis covered the period from 1993 (just after the launch of LAGEOS 2) up to 2004, including all SLR data from the two LAGEOS satellites. The data were reduced using 15-day orbital arcs with a one-day overlap. The models used were the most accurate and consistent with the IERS Conventions 2003. All known perturbations were modeled **except** for the LT effect (set to zero). Once all arcs were converged, for each LAGEOS we formed a time series of consecutive arcs' nodal longitude differences, i.e. the nodal longitude at  $t_d^{\text{ARC}=n+1}$  and the same quantity obtained for the same time from the previous arc at  $t_d^{\text{ARC}=n}$ . These were then integrated and combined using our constraint equation to generate a single time series. The secular trend of these series is the sought-for estimate of the  $\mu$  LT parameter. Figure 5 shows the final result for the 2004 experiment.



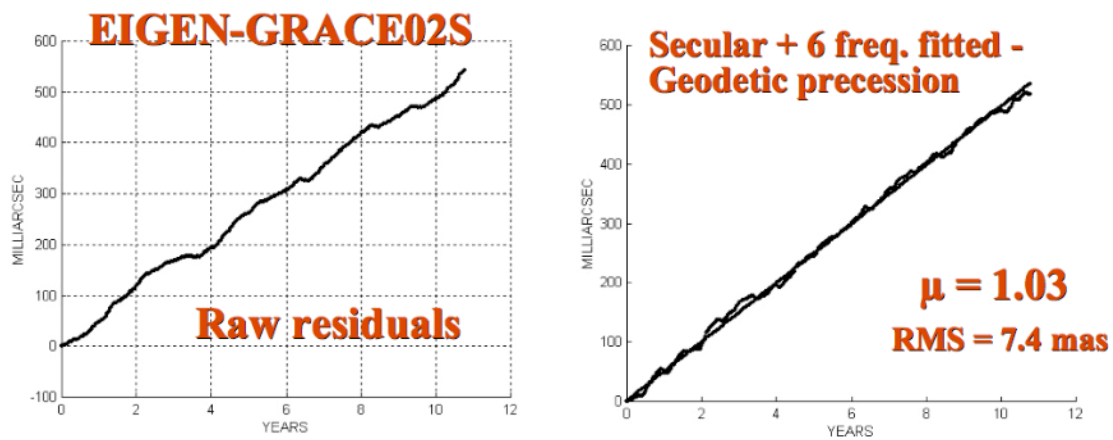
*Figure 5. The linear trend of the LAGEOS and LAGEOS 2 integrated nodal longitude differences time series for the EIGEN-GRACE02S gravitational model. Six periodic signals associated with well-known periods were filtered at the same time.*

We have already discussed the accuracy estimates associated with the 2004 result and the extensive work done to validate these error estimates as much as possible. It is worth noting that the gravitational model improvements from additional years of GRACE data result in an ever-improving estimate of these errors. The converging progression of these accuracy estimates provides a means to validate our quoted accuracy estimates for previous experiments. It is this point that makes the forthcoming new and much improved GRACE model GGM03S so anxiously awaited by all.

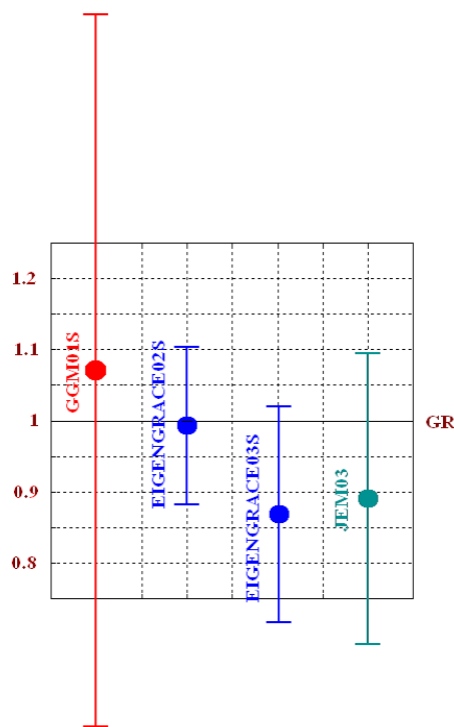
## Beyond the 2004 experiment

The LAGEOS experiments are a zero-budget verification experiment for the much more accurate ( $\sim 0.1\%$ ) and expensive ( $> \$700\text{M}$ ) result expected from NASA's Gravity Probe B mission (Buchman et al., 2000). In particular, with the recent

discovery of unanticipated errors in the gyro design of GP-B (Tomlin, 2007), it is doubtful that the GP-B results will ever break the 10% accuracy level (Kahn, 2007), so the LAGEOS experiments may eventually take a totally unforeseen center role in the area of fundamental physics tests.



**Figure 6.** Results from the GFZ software package EPOS, replicating the 2004 experiment (preliminary, pending small s/w improvements in the force model).



**Figure 7.** Results from the joint analysis for four different gravitational models from GRACE (plotted is the value of the recovered  $\mu$ , with unity signifying GR is correct).

To improve the validation of our results our original group was extended to encompass analysts from other institutions and allow an independent check of the results with multiple software packages and alternate reduction philosophy. So far, the GFZ group has become an integral and active participant with their software package EPOS. First results from their initial attempts to replicate our 2004 experiment are shown in Fig. 6. The small discrepancy with respect to our 2004 result is due to the

fact that their software needs some small improvements to match the modeling that was used in Geodyn. In addition to the test results for 2004, new models developed by various GRACE science team groups were also used to derive new estimates of  $\mu$ . Using different gravitational models we also get a good sense of the variability of the  $\mu$ -estimates due to the change in the model, the development group's strategy and their ability to properly calibrate the errors of their model. The results are shown in a summary plot in Figure 7.

### **LAGEOS results and LLR claims**

It is sometimes claimed that gravitomagnetism, measured already by SLR with the LAGEOS satellites, (might also be detected after refining the GP-B data analysis, see Tomlin, 2007), has already been observed by Lunar Laser Ranging (LLR), (Murphy in these proceedings and Murphy *et al.*, 2007); however the gravitomagnetic effects measured by LLR and the LAGEOS satellites are intrinsically different.

The gravitomagnetic effect measured by LLR depends on the motion of a gyroscope (the Earth-Moon system in the case of the LLR analysis) with respect to a central mass (the mass of the Sun in the LLR analysis) and, by changing the frame of reference used in the analysis, is equivalent to the geodetic precession, already well measured by LLR. The second gravitomagnetic effect measured by the LAGEOS satellites is an intrinsic gravitomagnetic effect (Ciufolini, 1994 and Ciufolini and Wheeler, 1995, Ciufolini 2007) that cannot be eliminated by means of any coordinate transformation.

In general relativity, in the frame in which a mass is at rest the so-called "magnetic" components  $g_{0i}$  of the metric are zero (in standard PPN coordinates). However, if an observer is moving with velocity  $\mathbf{v}$  relative to the mass, the "magnetic" components  $g_{0i}$  are no longer nonzero in his local frame. These "magnetic" components  $g_{0i}$  can be simply eliminated by a Lorentz transformation back to the original frame. This is precisely what has been observed by LLR since the first measurements of the geodetic precession of the lunar orbit. In contrast, a mass object (such as Earth) with angular momentum  $J$  generates a gravitomagnetic field intrinsic to the structure of spacetime that therefore cannot be eliminated by a simple coordinate transformation or choice of reference frame. This is the field producing the LT effect on Earth orbiting satellites such as LAGEOS, measured by SLR.

In general relativity, given explicitly a general metric  $\mathbf{g}$ , with or without magnetic components  $g_{0i}$ , in order to test for intrinsic gravitomagnetism (i.e. which cannot be eliminated with a coordinate transformation), one should use the Riemann curvature tensor  $\mathbf{R}$  and the spacetime invariants built using it (Ciufolini, 1994 and Ciufolini and Wheeler, 1995). Ciufolini and Wheeler (1995) give the explicit expression of the Riemann curvature invariant  $*\mathbf{R}\cdot\mathbf{R}$ , where  $*\mathbf{R}$  is the dual of  $\mathbf{R}$ . Irrespective of the frame of choice, this invariant is non-zero in the case of the Kerr metric generated by the angular momentum and the mass of a rotating body. When however we evaluate it for the Schwarzschild metric generated by the mass of a non-rotating body, it is equal to zero for any frame and coordinate system of choice. In (*ibid.*) it is shown that the gravitomagnetic effect measured by LAGEOS and LAGEOS 2, due to Earth's angular momentum, is intrinsic to the spacetime's curvature and cannot be eliminated by a simple change of frame of reference since the spacetime curvature invariant  $*\mathbf{R}\cdot\mathbf{R}$  is different from zero. However, the effect measured by LLR is just a gravitomagnetic effect that depends on the velocity of the Earth-Moon system and whose interpretation depends on the frame used in the analysis.

Murphy *et al.* (2007) show that on the lunar orbit there is a gravitomagnetic acceleration that changes the Earth-Moon distance by about 5 meters with monthly and semi-monthly periods. In a frame of reference co-moving with the Sun, the lunar gravitomagnetic acceleration in the Moon's equation of motion, is  $\sim \mathbf{v}_M \times (\mathbf{v}_E \times \mathbf{g}_{ME})$ ; where  $\mathbf{v}_M$  and  $\mathbf{v}_E$  are the velocities of Moon and Earth in the frame of reference co-moving with the Sun and  $\mathbf{g}_{ME}$  is the standard Newtonian acceleration vector on the Moon due to the Earth mass; this is the term discussed in (Murphy *et al.*, 2007). However, in a geocentric frame of reference co-moving with Earth, the lunar gravitomagnetic acceleration can be written:  $\sim \mathbf{v}_M \times (\mathbf{v}_S \times \mathbf{g}_{MS})$ : where  $\mathbf{v}_M$  and  $\mathbf{v}_S$  are the velocities of Moon and Sun in the frame of reference co-moving with Earth and  $\mathbf{g}_{MS}$  is the standard Newtonian acceleration vector on the Moon due to the Sun mass. This acceleration can be simply rewritten as a part equivalent to the geodetic precession (Ciufolini 2007) and another one too small to be measured at the present time.

This argument can be made rigorous by using the curvature invariant  $*\mathbf{R} \cdot \mathbf{R}$ . This invariant is formally similar to the invariant  $*\mathbf{F} \cdot \mathbf{F}$  equal to  $\mathbf{E} \cdot \mathbf{B}$  in electromagnetism. In the case of a point-mass metric generated by Earth and Sun, this invariant is:  $\sim \mathbf{G} \cdot \mathbf{H}$ , where  $\mathbf{G}$  is the standard Newtonian electric-like field of the Sun and Earth and  $\mathbf{H}$  the magnetic-like field of the Sun and Earth; this magnetic-like field is  $\sim \mathbf{v} \times \mathbf{G}$  and then clearly, on the ecliptic plane, the invariant  $*\mathbf{R} \cdot \mathbf{R}$  is null. Indeed, this invariant has been calculated (Ciufolini 2007) to be zero on the ecliptic plane, even after considering that the lunar orbit is slightly inclined on the ecliptic plane, this component would only give a contribution to the change of the radial distance too small to be measured at the present time.



**Figure 8.** A 1:2 model of the proposed LARES (Bosco *et al.*, 2006) geodetic satellite for SLR applications in relativistic tests and geodetic TRF development.

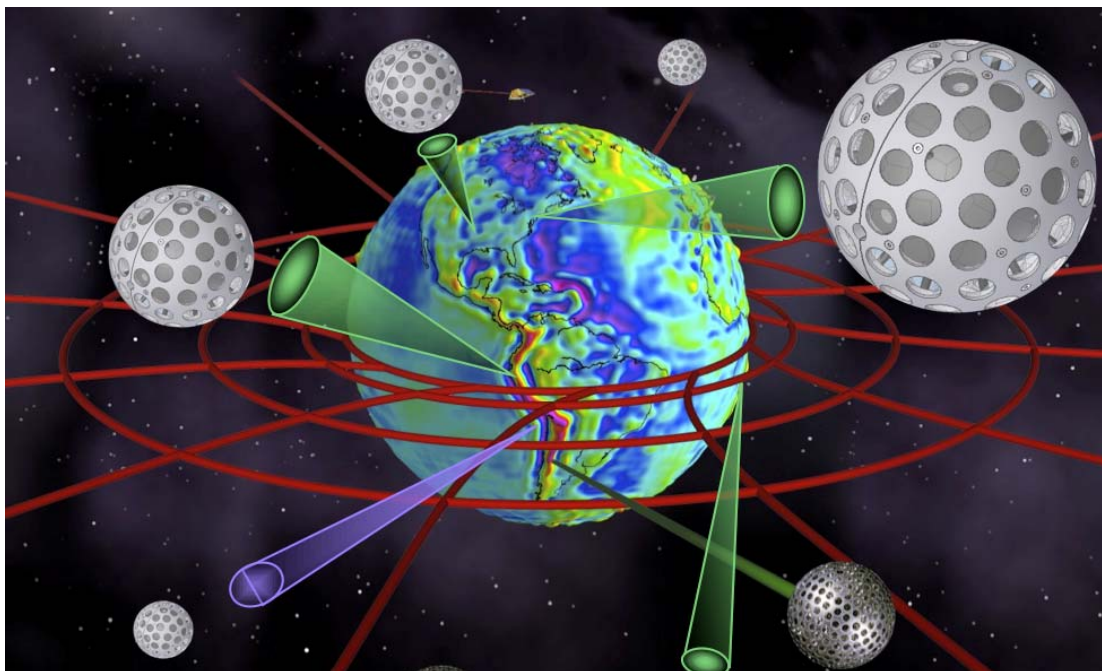


## Summary and future plans

The analysis of nearly twelve years of SLR data from LAGEOS and LAGEOS 2 has demonstrated the measurement of the LT effect at the 5-10% level for the first time. This result was possible because of the extremely precise gravitational models developed from the gravity-mapping missions CHAMP and GRACE. The results have been validated with independently developed s/w and our future plans include further additional validation with even more groups.

Interim results are also exchanged and compared with John Ries of Univ. of Texas, who is now using the UTEX software UTOPIA, in a similar reduction approach and obtains similar results. We hope to have UTOPIA results regularly in the near future, as the UTEX group makes time for participation in these experiments. It is our intention to have a new experiment using the new and soon to be released 3<sup>rd</sup>-generation UTEX model GGM03S, using all s/w packages (GEODYN, EPOS and UTOPIA) and groups, extending our LAGEOS data span by several years (3+) to the present, and incorporating many small but significant model improvements, especially in the temporally varying gravitational signals area due to climate change and global mass redistribution.

In a parallel process we are actively pursuing the optimal design and likely contribution of a new dedicated mission, LARES (Bosco *et al.*, 2006), which is currently in pre-phase B and expected to be in orbit in the next two years. Although not identical to LAGEOS, the improved design of LARES will result in a better LT measurement and expand the list of high-accuracy geodetic targets for TRF and low-degree temporal gravity observations. As explained in (*ibid.*), LARES is being designed with the utmost care for the definition of its “signature”, i.e. the precise offset between the effective reflection plane and its CoM, to minimize errors that affect the origin and scale of the TRF. A half-scale model of LARES is shown in



**Figure 9.** A visualization of the LT effect on frame coordinate lines and a constellation of geodetic satellite targets which with a small effort could be a reality by the end of this decade.

Figure 8 along with a mechanical drawing of the current design.

The future launch of LARES and other similar geodetic targets will go a long way towards the development of a “SLR” constellation (Figure 9). The near-continuous availability of targets at all SLR stations and the improved geometry from the mix of inclinations and nodal longitudes, etc., will lead to a more robust set of SLR products for TRF and POD. Improvement of the gravitational static and temporal models and the availability of other data sets from Earth observing missions will soon allow us to use most of the currently available and future geodetic satellites with laser arrays for highly precise geophysical products.

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